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PUBLIC DEVELOPMENT BANKS AND CREDIT MARKET IMPERFECTIONS

Abstract

This paper is devoted to understanding the role of public development banks in alleviating financial market imperfections. We explore two issues: 1) which types of firms should be optimally targeted by public financial support; and 2) what type of mechanism should be implemented in order to efficiently support the targeted firms' access to credit. We model firms that face moral hazard and banks that have a costly screening technology, which results in a limited access to credit for some firms. We show that a public development bank may alleviate the inefficiencies by lending to commercial banks at subsidized rates, targeting the firms that generate high added value. This may be implemented through subsidized ear-marked lending to the banks or through credit guarantees which we show to be equivalent in "normal times". Still, when banks are facing a liquidity shortage, lending is preferred, while when banks are undercapitalized, a credit guarantees program is best suited. This will imply that 1) there is no "one size fits all" intervention program and 2) that any intervention program should be fine-tuned to accommodate the characteristics of competition, collateral, liquidity and banks capitalization of each industry.

JEL Classification: H81, G20, G21, G23

Keywords: Public development banks, Governmental loans and guarantees, Costly screening, Credit rationing

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Public Development Banks and Credit Market Imperfections*

Marcela Eslava† and Xavier Freixas‡

March 7, 2016

Abstract

This paper analyses the role of a public development bank when banks use a costly screening technology to make credit decisions. We explore two issues: 1) which types of firms should be optimally targeted by public financial support; and 2) what type of mechanism should be implemented in order to efficiently support the targeted firms' access to credit. We show that, in the presence of costly screening, the market leads to an inefficient allocation, as there will be underprovision of credit. The market imperfection results from the inability of banks to appropriate the full benefits of projects they finance. This implies that the misallocation of credit is more pronounced for high value projects. This central result, and its implication that PDBs could play a central role in the financing of high value projects, contrast with the usual emphasis on credit underprovision for relatively weak projects/firms (SMEs, young firms, those without collateral, etc.). . We show that a public development bank may alleviate the inefficiencies by lending to commercial banks at subsidized rates and targeting the firms that generate high added value. This may be implemented through subsidized ear-marked lending to the banks or through credit guarantees which, in "normal times", we show to be equivalent . Still, when banks are facing a liquidity shortage, lending is preferred, while when banks are undercapitalized, a credit guarantees program is best suited to alleviate the constraints banks' face.

Keywords: Public development banks; governmental loans and guarantees; costly screening; credit rationing

JEL codes: H81, G20, G21, G23

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1 Introduction

The financing of firms by specialized public institutions is a pervasive feature of financial markets, whether in less developed economies, emerging or developed ones. Regional and global associations of development banks have over 280 members around the world, some of them large players in the credit markets of their respective countries.[1] The activities of these institutions are varied both in scope and focus. Some of them offer financing to a broad base of clients, while many others target particular types of firms, such as Small and Medium Enterprises (SMEs), startups, nascent or weak sectors (Figure 1). They also differ in the way they intervene: while some lend directly to businesses, others offer loans that are intermediated by private financial institutions (Figure 2). Many–73%, according to the Global Survey of Development Banks–offer public guarantees instead of, or in addition to, providing credit.

Still, it is not clear which financial frictions the provision of support by Public Development Banks (PDB) should be intended to remedy and which instrument, among those used by these institutions, is best suited for dealing with those frictions. Literature and practice have focused on financial market imperfections that imply credit underprovision for relatively weak projects/firms. Most PDBs, for instance, emphasize lending to SMEs (e.g. Figure 1). Theory has had a similar focus: PDB activity has been studied as a solution for the underprovision of credit for projects with negative low present value but positive externalities (Hainz and Hakenes, 2012); or for firms rationed out of credit for fear of moral hazard (Arping et al., 2010).[2]

In this paper, we study the role of a public development bank in the context of a model where banks use a costly screening technology to make credit decisions, and where they face at least some competition. The implication is credit underprovision resulting from the inability of banks to appropriate the full benefits of projects. High value projects are rationed out of credit because of this reason, leading to inefficient resource allocation. This central result, and its implication that PDBs could play a central role in the financing of high value projects, contrast with the usual emphasis on relatively weak projects.

Our model elaborates on standard costly information extraction, a major building block that has been developed to justify the existence of banks and their role in financial markets. It is thus a natural tool to study public interventions to deal with financial market imperfections. We use this theoretical framework

[1] Respondents of the World Bank´s Global Survey of Development Banks report participations in assets of between 9% and 19% in the respective market (Luna-Martínez and Vicente, 2012). Lazzarini et al (2014) report that the Brazilian Public Development Bank, BNDES, represents over 20% of loans in the Brazilian credit market, and amount to almost 10% of GDP.

[2] The theoretical literature on banking also provides a number of models where relatively weak firms will not have access to funding in spite of the fact that the project they want to finance has a positive net present value. This is the case of firms with a limited credit history (Diamond, 1991), lack of collateral (Holmstrom and Tirole, 1997, Ruckes, 2004) or, simply, risky (Bolton and Freixas, 2000). PDBs may play a role in alleviating financial imperfections in all of these contexts.

1

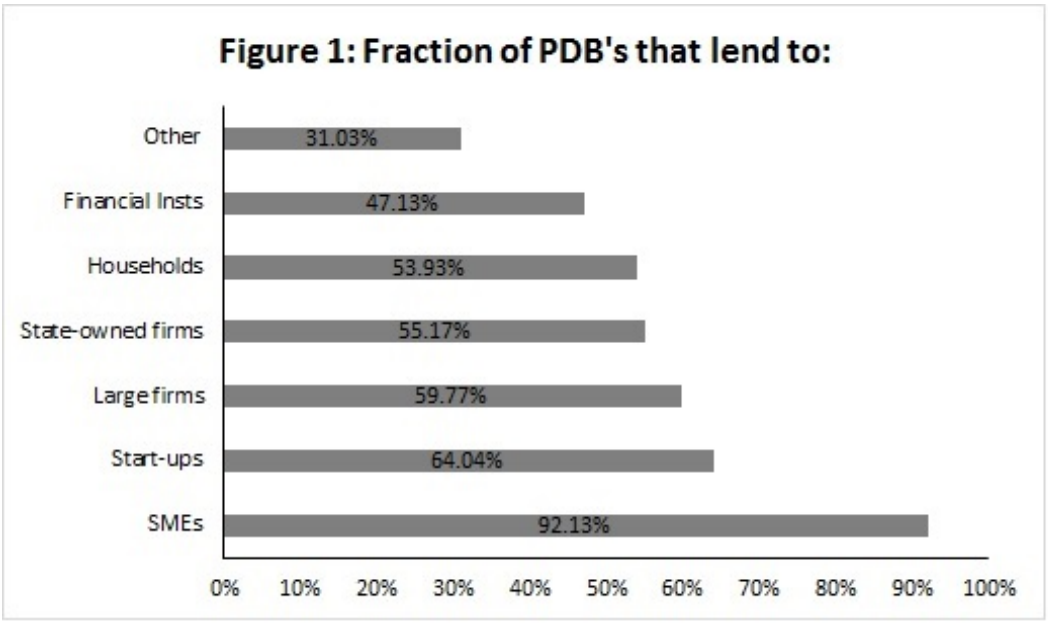
Figure 1: Fraction of PDB's that lend to:
Other
31.03%
Financial Insts
47.13%
Households
53.93%
State-owned firms
55.17%
Large firms
59.77%
Start-ups
64.04%
SMEs
92.13%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%

1.*jpg*

2

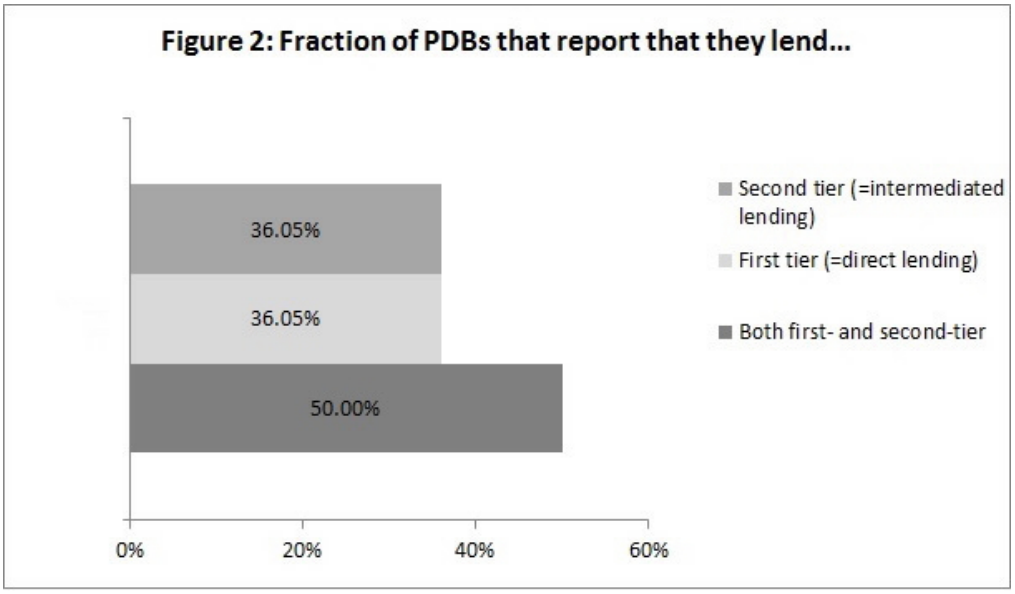
Figure 2: Fraction of PDBs that report that they lend...
36.05%
36.05%
50.00%
Second tier (=intermediated lending)
First tier (=direct lending)
Both first- and second-tier
0%
20%
40%
60%

2.*jpg*

to answer questions such as: 1) what types of firms, if any, should be the target of particular public support programs?; 2) should the public finance of firms take the form of direct or indirect lending? 3) if it takes the form of indirect lending, should the PDB lend to private banks at subsidized rates, or rather provide public guarantees (i.e. loss sharing)?

The model considers firms that require funding in order to implement their projects. Firms can be good or bad, and only good firms have positive net present value projects. They belong to "industries", which are characterized by a risk profile, so that "industries" may correspond to sectors or types of firms (young, SMEs,...). Certain "industries" may be characterized by higher net present value.

The type of a firm is not directly observable to either banks or the government. Still, commercial banks have access to a costly screening technology that yields a signal that may or may not be informative (Ruckes 2004). For any given firm, the bank and the firm will share the project's net present value. In equilibrium some good firms will be credit rationed, so that there is room for public intervention. The reason for the underprovision of screening is that banks do not take into account the externality they create when facilitating firms' access to credit and the rents they generate. The underprovision is more severe for types of firms where the rents the bank cannot appropriate are larger. The focus of our research is in evaluating the potential effectiveness of different instruments to deal with this credit market failure and to identify the types of firms that should be targeted.

We evaluate welfare, measured by net expected output, under alternative mechanism of public financial support, considering the effects of each mechanism on banks' behavior, as well as the implied costs of intervening. We derive the optimal conditions for subsidies to credit, as well as for public guarantees and direct government lending, and compare the relative merits of the different arrangements. Though direct government lending reduces distortions from taxation, indirect lending through the financial sector may turn out to be superior given potential political capture and failures in the corporate governance of the public development bank emphasized by previous literature. Within the indirect lending context, in turn, the optimal intervention depends on the returns and risk profiles of the projects that can benefit from the public policy. This has implications for the optimal targeting of government programs, including whether SMEs or other usual suspects are the best possible target.

The theoretical literature on banking also provides models in which credit rationing may arise as a consequence of moral hazard (Holmstrom and Tirole, 1997), or because of liquidity constraints in the financial market (Armendáriz, 1999). We extend our framework to consider these and other related issues and to shed light on how our core financial inefficiency interacts with others in shaping the optimal intervention for a PDB. In particular, we develop extensions to consider moral hazard, bank competition, use of collateral, liquidity and solvency restrictions, and business cycles. As a result, our paper also sheds light on the potential role of PDBs in these different contexts.

The empirical literature has shown that financing constraints affect more

starkly particular types of firms. For instance, SMEs report higher financing obstacles than large firms, and the effect of these financing constraints is stronger for them compared to more established firms (See Beck et al. (2008), Beck et al. (2005); Beck et al.,(2006) and Beck and Demirguc-Kunt, 2006 for an overview). Nevertheless, there is also heated debate about whether the more intense obstacles to growth SMEs seem to face indeed make them the optimal target of specific policies. For instance, while SMEs were for a long time seen as a main source for growth, Haltiwanger et al. (2013) show that the high growth believed to characterize small firms no longer correlates with firm size once firms' age is controlled for. These findings have been used to argue that targeting government support to young rather than small firms may be a better policy strategy. Our framework will contribute to this discussion, and related ones, by identifying features of firms that make them optimal target of policies aimed at alleviating credit rationing in our context.

In the next section we will describe our model and the financial market imperfection it implies, then turn to direct lending as a benchmark case. Section 3 will be devoted to a second best policy of subsidization to firms and banks. Section 4 will consider the impact of banks competition and its implications. Section 5 extends the analysis to explore the role of collateral, liquidity shortages and banks' capital shortages. Section 6 considers business cycles and Section 7 is devoted to the robustness of the qualitative results our framework delivers. Section 8 concludes.

2 The model

Consider an economy where all agents are risk neutral. Interest rates are normalized to zero. Different industries are characterized by risk parameters p, where p captures the potential probability of success of projects in the industry. Within industries, there are two types of firms, good and bad, in proportions μ and $1-\mu$. Good firms are at the industry's potential, facing probability of success p with an implied positive net present value, while bad firms have a lower probability of success p_-, yielding negative net present value. If successful, a project undertaken by a good firm yields an outcome of y per unit of investment, with constant returns to scale up to its full size I, so that a successful project of size I yields yI, while a null return is obtained if the project is unsuccessful ($yp > 1$ while $yp_- < 1$).[3]

The type (good or bad) of a firm is not observable to the bank or the government. The value of p, by contrast, is observed by the bank and by the public entity. The bank's role in the economy is to screen firms and, thus to weed out bad firms. We initially assume that banks' capital is not a constraint on their credit activity and address the solvency issues and the countercyclical role of PDBs, as well a moral hazard for firms, in extensions to the model.

[3] We do not rule out potential correlations between p, μ and y. To keep the exposition simple, however, our notation does not explicitly recognize these potential correlations.

2.1 Inefficiency in the market for credit

In order to fund their projects, firms approach banks that have a screening technology. For every industry/risk p, by paying a sunk cost $C(q)$, banks obtain a perfect signal on the firm's type good or bad (p or p_-) with probability q while, with probability $1-q$, they obtain no signal. We assume $C(q)$ satisfies $C'(q) > 0, C''(q) > 0, C(0) = 0$ and $C'(0) = 0$. If the bank receives a signal it will lend to good firms and deny credit to bad ones. If the bank does not receive a signal, we will assume it does not grant a loan, which occurs when μ is low (namely when $[\mu p + (1-\mu)p_-]\, y < 1$). We assume that screening costs are independent of the firm's project size. We justify our assumption because we associate an increase in size to higher complexity in the structure (balance sheet, multiple business lines,...), but this is compensated by a higher transparency. Because we assume the marginal screening cost is here relevant, to interpret our framework as our focusing on relationship lending, as we think lending based on credit scoring techniques is better characterized by a zero marginal screening cost.

The loan repayment per unit is $R(p) \leq y$, and depends upon the structure of competition in the credit market. We take $R(p)$ as given for the time being; a later section analyzes the setting of $R(p)$ and its implications for our central problem. We assume that all banks share the same technology, so that each of them obtains either the same signal or no signal. At this stage, we also assume that there is no collateral, an extension we consider later on.

Banks maximize their profits, choosing a level of screening for every type of risk (industry) p :

$$\max_{q(p)} \mu q(p)(pR(p) - 1)I - C(q(p))$$

This is a concave maximization problem with the following first order condition[4]

$$\begin{aligned} \mu\,(pR(p)-1)\,I &= C'(q(p)) \text{ for an interior solution} \\ \mu\,(pR(p)-1)\,I &> C'(1) \text{ for corner solution } q = 1 \end{aligned} \tag{1}$$

The case $\mu\,(pR(p)-1)\,I < C'(0)$ for corner solution $q = 0$ is excluded as we assumed $C'(0) = 0$

The screening level is thus increasing in the banks' return $pR(p)$, so that banks with higher market power will tend to finance more firms, even if they are riskier. This fact is in line with our model's results on bank competition, as a higher market power will imply a higher $pR(p)$ and this, in turn, will lead to a higher $q(p)$. In order to identify the financial market imperfections, it is useful to compare the market and the efficient allocation of credit. In doing so,

[4] The convexity of $C(q)$ jointly with $C(0) = 0$ allow us to dispense with the banks participation constraint, $\mu q(pR(p) - 1) \geq C(q)$. At the optimal point this constraint will always be satisfied.

we show that, in equilibrium, banks underprovide screening, with a consequent underprovision of credit in comparison with the efficient allocation level.

The efficient solution, in the perfect information case, results from the maximization of the aggregate output net of the production cost, where the central planner aggregates over industries (risk profiles):

$$\max_{q(p)} \int_0^1 \left[\mu q(p)(py-1)I - C(q(p))\right] dF(p)$$

The solution to this problem shows that the efficient level of screening is obtained for

$$\begin{aligned} \mu\,(py-1)\,I &= C'(q(p)) \text{ for an interior solution} \\ \mu\,(py-1)\,I &> C'(1) \text{ for a corner solution } q=1 \end{aligned} \tag{2}$$

We can now state our first result:

Proposition 1 *For levels of μ, such that $(\mu p+(1-\mu)p_-)y<1$, if $\mu\,(pR(p)-1)\,I < C'(1)$ market equilibrium leads to underprovision of screening by banks. The size of the inefficiency grows with $p(y-R(p))$, the rent the bank cannot appropriate.*
Proof. *Convexity of $C()$ and the feasibility condition $R(p)\leq y$ yield the result, by direct comparison of (1) and (2), except in the case where $\mu\,(pR(p)-1)\,I > C'(1)$, as $q=1$.* ■

The intuition behind Proposition 1 is simply that screening generates an externality: the good firm that is screened and obtains funding creates an additional output $y-R(p)$ with probability p, an expected profit the bank ignores[5]. So, the discrepancy between the market and the efficient provision of screening is precisely given by $\mu p(y-R(p))I$, the benefits that the bank does not fully internalize. Still, whether this inefficiency can be partially dealt with, and how, depends upon the instruments available to the government.

We now study some such instruments. As a benchmark, we first examine the problem of a PDB that directly lends to firms. Later, we solve the second best problem where the government, because of asymmetric information, moral hazard or imperfect corporate governance, cannot efficiently lend directly to firms, but is able to act as a principal and design mechanisms to support access to credit by subsidizing banks and firms activities.

2.2 The Direct Lending benchmark

The most straightforward way to channel credit to those firms that are credit rationed is to structure the PDB as a financial institution, with access to funding and equipped with a screening technology. This means the PDB directly lends

[5] Notice this is not due to the use of debt as the banks' financial instrument. Any other type of contract would generate the same effect, as the bank screening incentives would come from the from the fraction of the firm net expected profit it appropriates.

to firms. We study in this section a PDB that has access to the same screening technology that other banks have, but that may depart from maximizing the sum of total net output minus the cost of intervention, perhaps because of political considerations. Conventional wisdom is that the PDB could also (or alternatively) face higher screening costs than other banks due, for instance, to corporate governance failures. Though we do not model this possibility directly, our approach could be easily extended so as to capture different sources of inefficiency of direct government lending.

The simplest way to model the cost of intervention is to introduce a fixed distortion due to taxation. This is to be interpreted as the marginal cost of raising taxes when the tax scheme is optimal. Alternatively, the same parameter may reflect the shadow cost of the PDB budgetary restriction . Denote by λ the distortion associated to the raising of taxes to pay for the costs of government activities. Because the public bank obtains revenues $pR(p) - 1$ on each dollar lent, the cost of screening is an adjusted $[C(q(p))(1+\lambda) - \lambda\mu qI(pR(p)-1)]$. One benefit of direct lending by the PDB, compared to the market solution, is that the public institution internalizes the screening externality.

Departure of a PDB from maximizing net output minus the cost of subsidies is to be considered because of its potential lack of independence from politicians, because of imperfect corporate governance, limits to the remuneration policy and other characteristics of many public banks that may lead to a higher screening cost. Critics of public development banks (PDBs) worry that lending by these institutions may end up being inefficiently allocated due to political or institutional constraints. An abundant body of empirical evidence points at cases where this allocation seems to follow political considerations rather than seeking to maximize efficiency. Direct lending by PDBs has been found to increase in election years, and to be targeted to politically valuable costumers or regions, especially in election years (Carvalho, 2014; Cole, 2009; Dinc, 2005; Khwaje and Mian, 2005; Lazzarini et al, 2014; Sapienza, 2004).

We take into account the possibility that the PDB's agenda departs from strict welfare maximization by assuming that, instead of maximizing the net surplus $(py - 1)$, it maximizes a biased objective function $(p(y + \chi(p)) - 1)$. The generality of this formulation has the benefit of being open to a number of interpretations. Indeed, $\chi(p)$ (that we assume could be also be negative) may be interpreted as capturing measurement errors, institutional weakness, corruption or opportunistic behavior by politicians seeking election.

In our current formulation, the political rents $\chi(p)$ are lost if the project is not successful. Notice, nevertheless that, if we redefine $\chi'(p)$, as $\chi'(p) = p\,\chi(p)$, or $\chi'(p) = \mu p\,\chi(p)$ the alternative interpretation, of political rents unrelated to the success of the project, is obtained (Still, the extreme case of subsidies without screening in exchange for potential or actual campaign support is not covered). In what follows, we refer to the $\chi(p)$ bias as the "political economy drift", with the acknowledgement that alternative interpretations might fit better some environments than others.[6]

[6] Notice, nevertheless that this formulation do not cover cases of bribery, where by provid-

Assume, first, the political rents are such that

$(\mu p + (1-\mu)p_-)(y + \chi(p)) < 1$, so that without screening there would be no credit.

The PDB will then maximize:

$$\begin{aligned} \max_{q(p)} \int_0^1 & \{\mu q(p)\left[p(y+\chi(p))-1\right] I \\ & -\left[C(q(p))(1+\lambda) - \lambda\mu q(p) I(pR(p)-1)\right]\} f(p)dp \\ \text{s.t.} \quad & 1 \geq q(p); \end{aligned}$$

A simple way to see the objective function would be to provide the PDB with a mandate to maximize $\mu q(p)\left[p(y+\chi(p))-1\right] I$ and then to cover the cost $C(q(p))$ out of public funds that cost $1+\lambda$ per dollar. At the same time, as discussed above, the profit on the loan the firm pays to the PDB is an income to the Treasury and thus has a social benefit of $\lambda\mu q(p) I(pR(p)-1)$.

Denoting by $\delta(p)$ the Lagrangian multiplier associated with $1 \geq q(p)$, the first order condition with respect to $q(p)$ is given by:

$$\mu I\left[p(y+\chi(p)) - 1 + \lambda(pR(p)-1)\right] - C'(q(p))(1+\lambda) - \frac{\delta(p)}{f(p)} = 0 \qquad (3)$$

Abstracting first from political rents, it is clear that direct lending by the PDB increases screening with respect to the market solution, and subsequently increases lending, bringing them closer to the first best solution. In particular, with direct PDB lending, and focusing for simplicity in the case with $q < 1$, condition (3) implies that the equilibrium screening level will be characterized by:

$$\mu I(py-1) = C'(q(p)) + \lambda\left[C'(q(p)) - \mu I\left(pR(p)-1\right)\right] \qquad (4)$$

The left hand side of this equation highlights the fact that the PDB fully internalizes the benefits of funding the positive value projects, while the last term of the right hand side captures the cost of the intervention compared to the first best. If λ were zero, meanwhile, this equilibrium condition would yield the first best level of screening. With $\lambda > 0$, however, the equilibrium implies a level of screening lower than the optimal. To see that this is the case, note that the term $\left[C'(q(p)) - \mu I\left(pR(p)-1\right)\right]$ is positive when evaluated at the first best q, so that a lower level of q is necessary to satisfy condition (4).[7]

This equilibrium implies higher q than the market solution, but, because λ is strictly positive, the first best cannot be reached. If, in turn, the cost of intervening is high enough that q under intervention is lower than in the market solution, the PDB should abstain from intervening.

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ing a subsidy to a firm the politician obtains a kickback, which would imply $\chi(p) = G(S_F(p))$, where G is an increasing function. Instead, the λ cost of any subsidy is accounted for, independently of the political drift $\chi(p)$.

[7] The first best satisfies $C'(q) = \mu I(py-1) > \mu I(pR-1)$.

Even if λ is low, however, PDB intervention can do more harm than good in the presence of what we have called political rents. It is clear that these rents will lead to two biases with respect to the optimal policy in the $\chi(p) = 0$ benchmark case. Regarding the level of screening, expression (3) states that a positive $\chi(p)$ will lead to an excess of screening $q(p, \chi(p))$ while a negative value for $\chi(p)$ will lead to an underprovision of screening.

Consider now the extreme case where $(\mu p + (1-\mu)p_-)(y+\chi(p)) > 1$. Then, because the firm will always end up being financed, the PDB will choose not to screen and all "bad" projects, in proportion $1-\mu$ will be financed. That is, even lending to bad firms may yield political benefits that make it attractive to the PDB, leading to inefficient lending. Interestingly, whether this holds or not depends on the industry's p. Certain industries or types of firms may yield particularly high political rents to the politician. The implication is an additional source for inefficiency: the credit allocation is distorted towards these politically attractive groups of firms.

Notice the question of whether PDB lending is a substitute or a complement of commercial banks' activity only makes sense when referring to direct lending, as credit guarantees or intermediated lending will only complement banking activity. In our framework, the answer to this question is straightforward: when it comes to direct lending, the activity of the PDB is a substitute and directly competes with commercial bank lending in so far as it lends to firms that generate a sufficiently high pledgeable cash flow; it is a complement when we introduce moral hazard at the firms' level and only for firms that receive a subsidized loan because of the moral hazard it faces, in which case they would never have been financed by banks. A different issue is whether the implementation of subsidized lines of credit to banks by the PDB will substitute market funding. As we will see below this will indeed be the case, although it will not be in competition with market funding and it will increase banks' profit, so that claims of unfair competition are not supported by this analysis.

3 Second best

An alternative to direct lending by the PDB is public lending intermediated by a private financial institution. The benefit of indirect lending is that it limits the political drift that may be inherent to direct lending. There are several reasons why this is so:

- Lending occurs only if the banks deem it profitable
- Firms are selected by banks, not by the PDB
- The lending or credit guarantees programs do not target specific firms but specific characteristics

Intermediated lending may be subsidized, or not, depending on the conditions banks and firms face. We will now assume the government is able to

subsidize the credit activity of banks, and determine to what extent and under which conditions it is optimal to set positive subsidies[8]. We will denote by $S_C(p)$ the per dollar loan credit subsidy, so that the total cost of the subsidies to lending to industry p will be $\lambda\mu qIS_C(p)$.

We assume the industry characteristics, p, and subsequently y and $R(p)$ are observable. It is thus possible to implement a policy of credit subsidies that are industry (or risk) dependent. As it is obvious, unconditional subsidies will not affect the agents behavior and, consequently, we directly consider subsidies that are related to the granting of a loan. The assumption that p is equally observable to both banks and the government is a useful starting point, but we later discuss the implications of relaxing it to address questions of central interest, such as how does optimal public intervention change when the bank has better information than the government about its client firms.

Consider the problem with λ, being, as before, the distortion associated to the raising of taxes. This approach may overestimate the cost of the subsidies as the profit the bank obtains from the subsidy will, presumably, be subject to taxation.

$$\begin{aligned}
&\max_{S_C(p),q(p)} \int_0^1 \left[\mu q(p)(py-1)I - C(q(p)) - \lambda\mu q(p) I(S_C(p))\right] f(p)dp \\
&\mu(pR(p) + S_C(p) - 1)I - C'(q(p)) \geq 0 \qquad (5) \\
&S_C(p) \geq 0; \qquad 1 \geq q(p);
\end{aligned}$$

Where constraint (5) holds with equality because the government should subsidize only up to the point where the bank is just induced to provide the second best's q.

Denote by $\nu(p)$ the Lagrangian multiplier associated to constraint (5), and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_C(p)$, and $q(p)$ are:

$$\begin{aligned}
-\lambda q f(p) + \nu(p) &\leq 0 \qquad (6) \\
\mu I\left[py - 1 - \lambda S_C(p)\right] - C'(q(p)) & \qquad (7) \\
-\frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} &= 0
\end{aligned}$$

We now examine the optimal $S_C(p)$ for a sector characterized by p.

To begin with, notice that since there is no inefficiency when the market leads to full screening ($q = 1$), there is no point in subsidizing the bank when this is the case. The proof is straightforward, because for $q = 1$ constraint (5) is not binding, and consequently, $\nu(p) = 0$, but then condition (6) holds with a strict inequality, which implies $S_B = 0$.

[8] Of course, subsidizing banks may imply that the tax structure should be rearranged. If so banks may receive a subsidy on their lending activity while taxed on their profits.

Focusing now on the interior solution for S_C, such that $S_C > 0$ and (6) holds with equality, we can replace (6) into (7) to obtain:

$$\begin{aligned} \mu I \left[py - 1 - \lambda S_C \left(p \right) \right] - C'(q\left(p \right)) & \\ -\lambda q C''(q(p)) - \frac{\delta(p)}{f(p)} &= 0 \end{aligned}$$

Substracting constraint (5) we obtain:

$$\mu(p(y - R(p)) - S_C\left(p \right)) - \mu\lambda S_C(p)) - \lambda q \frac{C''(q(p))}{I} - \frac{\delta(p)}{If(p)} = 0 \tag{8}$$

We thus have

$$S_C(p) = \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(q(p)) \right] \frac{1}{1+\lambda} \text{ if } q(p) < 1 \tag{9}$$

$$S_C\left(p \right) < \left[p(y - R(p)) - \frac{\lambda}{\mu I} C''(1) \right] \frac{1}{1+\lambda} \text{ if } q(p) = 1 \tag{10}$$

where $S_C(p)$ and $q(p)$ satisfy (5).

The interior solution (9) subsidy $S_C(p)$ compensates for the upside the bank ignores when it takes its screening decision, so it depends upon $y - R(p)$. Derivation of expression (5) with respect to q yields the marginal cost of driving q up via subsidizing the bank, given by $\frac{\partial S_C}{\partial q} = \frac{C''}{\mu I}$. The level of the subsidy is greater the lower is this marginal cost (see (9)), the larger the distortionary cost of taxation λ, and the lower the optimal subsidy . Notice that the second best $q(p)$ will always be lower than the first best because $\lambda > 0$. If there was no distortion associated to the use of fiscal revenue, $\lambda = 0$, then the first best would, obviously, be obtained.[9]

Proposition 2 *The second best efficient solution requires to set a subsidy to bank lending that is increasing in the externality associated with banks screening; decreasing in $C''(q(p))$ and decreasing in the distortions associated with using fiscal resources, λ; and increasing in μ and p.*

To understand the implications of this result in terms of the industries that should be targeted, notice that condition $S_C(p) > 0$, together with (9) and (10), imply

$$p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p)) \tag{11}$$

That is, subsidies are granted to banks on loans directed to p profiles for which the externality is stronger and the cost of subsidizing is smaller, where the latter happens for larger projects and when the second best q is sufficiently

[9] The distortion λ plays a key role later in our discussion of the merits of direct vs. intermediated public funding.

high. It is also the case that subsidizing banks is optimal when the probability of finding a good project is sufficiently high and the cost of taxation sufficiently low.

Proposition 3 *In the second best solution the bank will be subsidized for loans to firms that satisfy* $p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p))$, *where* $q(p) = Min\{1, q^*\}$ *and* q^* *is the solution*[10] *to:* $\mu I \left\{ pR(p) + \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(q(p)) \right] \frac{1}{1+\lambda} - 1 \right\}) = C'(q(p))$.

Notice that intervention may optimally bring banks to fully screen in cases where the market solution implies $q < 1$. To fulfill constraint (5) the government need only provide S_C up to the point that satisfies:

$$\mu I(pR(p) + S_C(p) - 1) = C'(1) \tag{12}$$

implying $\mu(pR(p) - 1)I - C'(1) < 0$. That is, the banks would not have reached the $q(p) = 1$ level without the subsidy.

In the previous analysis, the optimal choice of S_C has only been implicitly solved for, as condition (9) defines S_C in terms of endogenous variable $q(p)$. As an example with a closed form solution, suppose that $C(q) = \alpha q$. The market allocation would be:

$$q = 1 \text{ for } pR(p) - 1 > \frac{\alpha}{\mu I} \tag{13}$$

$$q = 0 \text{ for } pR(p) - 1 < \frac{\alpha}{\mu I} \tag{14}$$

If (13) holds, then the government would abstain from subsidizing the bank ($S_C = 0$). But if instead we have a corner market solution without screening (14), the government might offer a subsidy just enough to bring the bank to fully screen:

$$S_C(p) = \frac{\alpha}{\mu I} - pR(p) + 1$$

For this subsidy to be optimal, however, condition (10) should hold, so that the social cost of the subsidy has to be lower or equal to the social benefit, $p(y(p) - R(p))$

$$S_C(p)(1 + \lambda) \leq p(y(p) - R(p))$$

Replacing $S_C(p)$ we obtain the locus of industries for which it is efficient to subsidize:

$$p(y - R(p)) \geq \left\{ \frac{\alpha}{\mu I} - (pR(p) - 1) \right\} (1 + \lambda)$$

[10] We are assuming a solution exists, which is generally the case. There are exceptions, one of which is simply the linear cost function where $q(p)$ will neither appear in the left hand side nor in the right hand side.

These are industries for which the value of the externality being addressed $p(y - R(p))$ exceeds the (net) cost of screening, adjusted by the distortionary cost of taxing. Suppose for instance that banks obtain a constant markup m over each loan (so R decreases with p to guarantee that $(pR(p) - 1)$ is constant). This last expression makes it clear that, conditional on $pR(p) - 1 < \frac{\alpha}{\mu I}$, so that market screening is not perfect, only high upside value industries (high p, high y) are subsidized, because it is for them that the externality to be addressed is largest, with a constant subsidy $S_C = \frac{\alpha}{\mu I} - m$. High banking competition reduces the markup for banks, increasing the size of optimal subsidies to incentivize screening. This discussion, as noted, holds if $pR(p) - 1 < \frac{\alpha}{\mu I}$, so that decreased risk (increased p) makes subsidies more likely only for cases in which risk and screening costs are not so low that the market already provides perfect screening.

The quadratic cost function case provides another useful illustration. Assuming $C(q) = \frac{1}{2}\beta q^2$, it is also easy to solve for $q(p)$ and $S_C(p)$:

$$\begin{aligned} S_C(p) &= \frac{p(y - R(p)) - \lambda(pR(p) - 1)}{1 + 2\lambda} \\ q(p) &= \frac{\mu I}{\beta}\left[\frac{py - 1 + \lambda(pR(p) - 1)}{1 + 2\lambda}\right] \end{aligned} \tag{15}$$

The parameter constellations for which $S_C(p) > 0$ are illustrated in Figure 3. Consistent with our discussion of the linear case, the optimal subsidy is positive if p is large enough, and the size of the subsidy also increases with p. The range of p's satisfying this condition expands if y is higher (light grey line rather than black line). In turn, higher distortionary costs of taxation, lambda, reduce the range of subsidized p's and the size of the subsidy (dashed line).[11]

3.1 Economic Interpretation

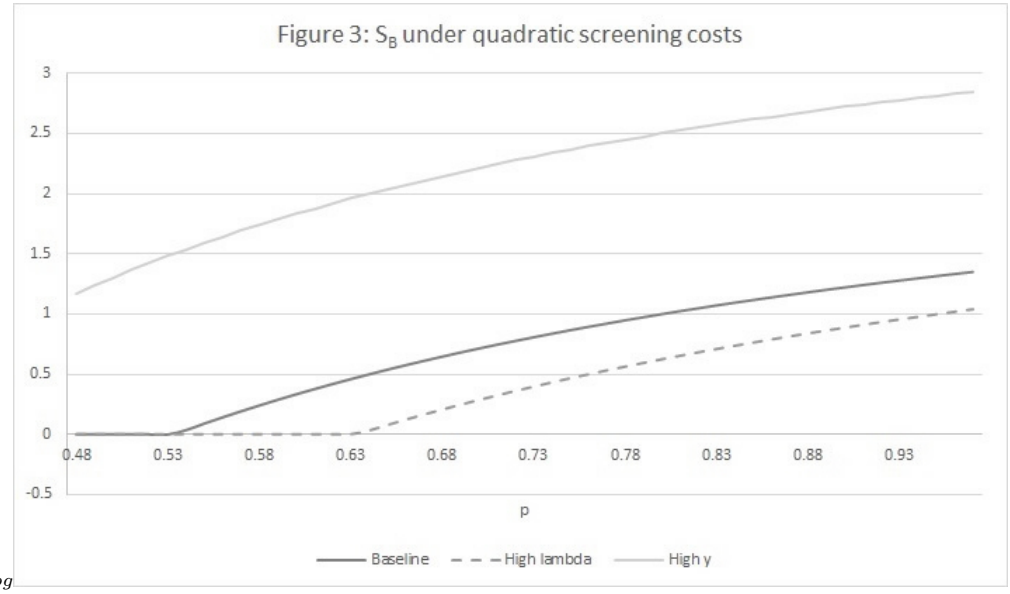
Our setup highlights the central role of the externality that leads to screening underprovision: financiers do not fully internalize the benefits of lending because they cannot appropriate them (i.e. $y - R(p) > 0$), and thus put less effort than it would be optimal in obtaining a precise signal about a potential costumer. By pinpointing this specific market failure, the analysis makes clear that a subsidy to banks, conditional on their granting a loan, is a natural intervention. The analysis allows us to clarify which types of firms/loans should be targeted.

In particular, condition (11) implies that the subsidies S_C should target industries characterized by:

$\mu p(y - R(p)) \geq \frac{\lambda q}{I} C''(q(p))$ (condition 11), but only as long as $\mu I(pR(p) - 1) - C'(1) < 0$,(eq. 12) so that $q(p) < 1$ in absence of the subsidy. This implies the targeted firms are characterized by:

[11] To generate this figure, we choose parameter values that satisfy the modelling assumption that $p\mu y > 1$. In particular, we assume $\mu = 0.7$; $y = 3$ in the baseline and $y = 4.5$ in the high y case; and $p \geq 0.48$.

3



3.jpg

1. Sufficiently high expected firms' profits (i.e. high $\mu p\left(y - R\left(p\right)\right)$) as this reflects the inefficiency of credit rationing the subsidies intend to remedy, proportional to the benefits not internalized by the bank. It is important to notice that our analysis characterises the second best for every level of y and μ. The whole analysis carries over to any dependence of y and μ on the industry characteristics, p. As a consequence, it is valid for any level of correlation between p and y on the one hand and between p and μ on the other hand.

2. Industries/types of firms for which the markup that banks obtain on loans are low (low $pR(p) - 1$), as these are the industries where the market q may be below 1, and therefore screening is underprovided. Fierce bank competition may, therefore, justify bank subsidies S_C.

3. Though we have so far assumed that screening costs are not industry-specific, if these costs were to vary across industries our analysis would suggest that subsidies should be directed only to industries/clients for which the marginal screening cost is sufficiently high for there to be screening underprovision.

4. Projects with sufficiently large financing needs I and with a high proportion of good firms μ .

Some of these implications challenge the conventional wisdom about valid targets for the public financing of enterprises. Credit for firms/projects with high expected returns is frequently deemed unworthy of subsidizing, under the

expectation that they will be particularly well served by the market. Our results, however, point out that these projects may be, in fact, the ones where subsidies will be more effective. Low risk (high p) and high y industries are, in consequence, plausible targets of S_C, except in the extreme case where their risk is sufficiently low that the market would grant $q = 1$ without subsidies.

Our results also make it clear that subsidizing loans for large projects/firms may in fact be optimal given the large expected benefits of these loans. Loans to sectors facing particularly dynamic demand growth, or those to firms with risky but high upside value projects, are plausible targets of this policy.

It is also clear from these results that external positive effects on other firms (other than the one receiving the loan), often deemed as the justification behind the government financing of enterprises, are not a necessary condition for subsidies to be optimal. Even in their absence, the fact that the financier cannot fully internalize the benefits of lending leads to loan underprovision. Of course, when externalities over third firms are in fact present they represent an additional reason for an intervention that subsidizes loans, a point that is beyond the scope of our model.

3.2 Implementation: Subsidized Lending vs. Credit Guarantees Programs

To begin with, notice that the cost of subsidizing banks has only been taken into account, and if we take the natural interpretation of λ as the cost of tax distortions, once the optimal tax scheme is implemented it may imply an additional adjustment in the banks' tax treatment, so that it may be optimal to increase banks' taxes on profits to compensate for the increase in profits the subsidy implies.

The simplest interpretation for the credit subsidy is as a direct subsidy $S_C(p)$, per dollar of loan, which makes it conditional on the loan being granted. This is not the most usual practice, however. The effects of the subsidy could be reached alternatively by funding credit in conditions that entail an implicit subsidy to the credit activity. Since what is relevant here is to reach a level of $pR(p) + S_C(p) - 1$ that would lead banks to increase their level of screening to the second best level, a policy of subsidized funding to banks at below the market rate, $1 - \delta$, will lead to the same result provided the terms of the loan, $R(p)$, are agreed beforehand[12]. Otherwise cheaper funding might simply be a source of rents for the bank. By setting $\delta = S_C(p)$, the second best allocation will be reached.

Alternatively a policy of credit guarantees will also allow to reach the second best allocation, although with a strong limitation imposed by the banks' incentives to screen. Indeed, too generous a credit guarantee policy would lead the bank to prefer lending to the average firm and save on the screening costs.

[12] This issue is more involved if $R(p)$ is the equilibrium lending rate, resulting from competition. Indeed, the subsidy may prove ineffective if, because of competition, it is passed down to firms.

A credit guarantees policy will be defined by a payment of an amount $G(p)$ to the bank in case the firm defaults. So, under this scheme, the bank receives $G(p)$ for every firm that defaults.

A bank will prefer to screen firms rather than to lend to the average firm provided:

$$\mu q(p)(pR(p) + (1-p)G(p) - 1)I - C(q(p)) \geq \tag{16}$$
$$\mu(pR(p) + (1-p)G(p) - 1)I + (1-\mu)(p_-R(p) + (1-p_-)G(p) - 1)I \tag{17}$$

We will assume this condition is satisfied, which implies $G(p)$ is lower than some threshold $\overline{G(p)}$. In terms of the bank incentives, credit guarantees means that the bank return on a loan will be $pR(p) + (1-p)G(p) - 1$. Consequently, the credit subsidy can be implemented in this way[13], by setting $G(p)$ so that $(1-p)G(p) = S_C(p)$, or $G(p) = \frac{S_C(p)}{1-p}$.

Notice that in our framework there is no difference between a credit guarantees program and selling a credit default swap at the subsidized price.

To summarize, a credit support policy program should

1. Develop the information available to the PDB, so that it has the best possible information on industries characteristics and screening costs, while banks have an efficient screening procedure (a credit registry greatly reduces screening costs).

2. The PDB should identify the level of credit rationing in each industry, which measures q, and disregard industries with no credit rationing ($q = 1$).

3. The PDB should identify the industries with the higher upside potential $p(y - R(p)$ that are facing credit rationing.

4. The PDB should determine the marginal impact of a subsidy on the banks screening level (that depends upon competition and the shape of the screening cost function $C''(q)$.

3.3 Comparing direct government lending vs. second best

In order to compare direct government lending and the second best allocation obtained through subsidies, it is useful to consider, first, the hypothetical case of an unbiased public bank, that is with $\chi = 0$. In this case, it is easy to see that direct lending is superior to the second best allocation.

[13] Notice that a policy where the PDB charges banks for the credit guarantee is also possible. In such a case, if the PDB charges an amount γ per dollar of loan for the guarantee $G(p)$, the subsidy would be implemented by setting $(1-p)G(p) - \gamma = S_B(p)$, so that, provided condition (16) is satisfied, charging for the credit guarantee allows to extend their amount.

Remark 4 *When direct lending is unbiased ($\chi = 0$), it is superior to the second best allocation. This is the case because the value of the objective function is higher, as the cost of intervention is reduced because $pR(p) - 1 - C(q(p)) > 0 > -\mu q S_C$ while the direct lending maximization problem feasibility set is larger as the bank incentive constraint is dropped from the program.*

By continuity, the previous remark implies that for small levels of political drift χ, direct lending is preferred, while for larger levels the indirect intervention through the subsidization of banks and firms will be preferred.

Nevertheless, the empirical evidence already mentioned seems to suggest that, at least for some PDBs, the bias χ is quite significant. One possible reason is that the PDB does not have access to the same information about risk profiles p than private banks. Another is institutional weakness leading to the direct lending process not being autonomous with respect to the government and the objectives and constraints of its leaders. Others are corruption and more stringent legal constraints that bind public agencies compared to private institutions. In contexts where any of these reasons weigh sufficiently, the distortion that χ brings to PDB lending outweights its benefits and the implementation of a direct lending program by the PDB will be inefficient.

3.4 Moral hazard

Suppose now that firms may engage in moral hazard behavior, as in Holström and Tirole (1997), which, as we will see, may lead to additional underprovision of credit. In particular, firms are able to choose a project that yields private benefits B at the expense of a lower probability of success, $p - \Delta p$ or $p_- - \Delta p$. Projects by firms that engage in moral hazard behavior yield a negative expected return, even if the firm is good: $(p - \Delta p)y < 1$ (and a fortiori $(p_- - \Delta p)\, y < 1$).

For a given repayment $R(p)$, the firm will choose the high probability of success project, rather than enjoying the private benefits if and only if:

$$p(y - R(p))I \geq (p - \Delta p)(y - R(p))I + B \tag{18}$$

that is,

$$R(p) \leq y - \frac{B}{I\Delta p} \tag{19}$$

In other words, for firms to avoid engaging in moral hazard behavior, banks must leave a sufficient rent to the firm. The maximum repayment must be in line with the pledgeable income $y' \equiv y - \frac{B}{I\Delta p}$. For the sake of simplicity, we will assume condition (19) is satisfied for any p larger than some floor level $\underline{p}$, but a different set of assumptions could be made with only minor impact in the formal analysis.

When condition (19) is satisfied, the moral hazard constraint is not binding and the above results of the costly screening problem developed in the previous sections simply applies. Yet, firms with p outside this range will be rationed out of credit for any given level of the repayment. This opens additional room for public intervention.

Consider a modified second best problem where the PDB can provide a performance premium to firms, to provide incentives for firms to behave. Denote this performance premium by $P_F(p)$. The PDB problem is now:

$$\begin{aligned}
&\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \left[\mu q(p)(py-1)I - C(q(p)) - \lambda\mu q(p) I(S_C(p)+pP_F(p))\right] f(p)dp \\
&\mu(pR(p)+S_B(p)-1)I - C'(q(p)) = 0 \\
&\left[y + P_F(p) - R(p)\right] I \geq \frac{B}{\Delta p} \qquad (20) \\
&S_B(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p);
\end{aligned}$$

Denote by $\gamma(p)$ the Lagrangian multipliers associated to constraint (20), and let $v(p)$ and $\delta(p)$ continue to be the multiplier associated, respectively, with the private bank constraint and the positivity constraint for $q(p)$.

The first order conditions with respect to $S_B(p), P_F(p), q(p)$ and p^* are:

$$\begin{aligned}
-\lambda q f(p) + \nu(p) &\leq 0 \\
-\lambda\mu p q(p) f(p) + \gamma(p) &\leq 0 \qquad (21) \\
\mu I\left[py - 1 - \lambda\left(S_B(p) + pS_F(p)\right)\right] - C'(q(p)) - & \\
-\frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} &= 0 \\
\left[\mu q(p^*) I\left[p^* y(p^*) - 1 - \lambda(S_B(p^*) + pS_F(p^*))\right] - C(q(p^*))\right] &= 0 \qquad (22)
\end{aligned}$$

Setting $S_F(p) = 0$ is optimal if $p > \underline{p}$, where $\underline{p}$ is the unique solution to $[y - R(p)] I(p)\Delta p = B$. The government optimally sets $S_F(p) = 0$ when $p > \underline{p}$, because in this region constraint (19) holds with strict inequality for any level of S_F.

If $p < \underline{p}$, meanwhile, a positive subsidy to the firm ($P_F(p) > 0$) opens up as a possibility. $P_F(p) = 0$ would still be optimal, however, in the specific case where $q(\underline{p}) = 0$. In this case, the sector (p) that just fulfills the no moral hazard condition would not be granted credit. Subsidizing sectors with p even marginally below $\underline{p}$ so that they abstain from moral hazard behavior is worthless, as moral hazard is not the reason why they are not granted credit. If the opposite holds, that is if $q(\underline{p}) > 0$, then it is optimal for the government to subsidize firms so that they behave, which in turns makes them credit worthy. This is stated in the following Proposition.

Proposition 5 *If $C(q)$ is strictly concave and banks make positive profits in industry $\underline{p}$ ($\underline{p}R(\underline{p}) + S_C(\underline{p}) > 1$), then $p^* < \underline{p}$*

Proof. $P_F(\underline{p}) = 0$, as otherwise it would imply a strict inequality $[y + P_F(p) - R(p)] I(p)\Delta p > B$. Plugging $P_F(\underline{p}) = 0$ into the first order condition (22) implies:

$$\mu I\left[\underline{p}y(\underline{p}) - 1 - \lambda S_C(\underline{p})\right] = C'(q(\underline{p})) + \lambda q C''(q(\underline{p}))$$

On the other hand, maximization with respect to p^* implies:

$$\mu I(p^* y(p^*) - 1 - \lambda(S_C(p^*))) = \frac{C(q^*(p^*))}{q^*(p^*)}$$

Because $pR(p) + S_C(p) > 1$ we have $q\left(\underline{p}\right) > 0$ and the strict concavity of $C(q)$ implies $C'(q\left(\underline{p}\right)) + \lambda q C''(q(\underline{p})) > \frac{C(q^*(p^*))}{q^*(p^*)}$. Thus, we have $\underline{p} \neq p^*$, implying $p^* < \underline{p}$. ■

The above proposition states that, as long as $q(\underline{p}) > 0$, there is always a fringe of firms that it is worth targeting with performance premia (a range of $p < \underline{p}$ such that $P_F > 0$). The intuition is simply that a very small premium will allow the firm to be financed (provided, of course $q(p) > 0$, for which a sufficient condition is the existence of positive profits for the bank) and this will bring an increase in both banks and firms' profits.

If the premium P_F is positive, then $\gamma(p) = \lambda \mu p q(p) I f(p)$ and this implies the associated constraint is binding, so that

$$P_F(p) = \frac{B}{\Delta p} - y + R(p)$$

The positivity of $P_F(p)$ implies that subsidies go only to firms that would not have been financed otherwise, as their profit $y - R(p)$ would be lower than $\frac{B}{\Delta p}$. That is, $P_F(p) > 0$ only for $p \epsilon (p^*, \underline{p})$

Now, using (8), we derive the value for $S_C(p)$

$$\mu(p(y - R(p)) - S_C\left(p\right)) - \mu\lambda(S_C(p) + pP_F(p)) - \lambda q \frac{C''(q(p))}{I} - \frac{\delta(p)}{f(p)} = 0$$

implying, for the interior solution:

$$S_C(p) = \left[p(y - R(p)) - \lambda p P_F(p) - \frac{\lambda q}{\mu I} C''(q(p))\right] \frac{1}{1 + \lambda}$$

Consequently, there is some trade-off between the two subsidies, as each dollar of additional subsidy to the firm leads to a decrease of $\frac{\mu\lambda}{1+\lambda}$ in the bank's subsidy. The intuition is obvious: because a subsidy to a firm creates a distortion $\mu\lambda p P_F(p)$, this comes as a reduction in the benefits $p(y - R(p))$ from a banking subsidy.

The analysis in this section shows that premia to ex post successful firms may be and optimal complement to credit subsidies, but only for relatively high risk firms ($p < \underline{p}$), as a way to reduce the moral hazard incentives that prevent these firms from accessing credit. Notice also that a pure subsidy to a firm, unconditional on the success of the project would have no effect on the moral hazard constraint, as it would be added both to the left and right hand side of condition (18).

The conditional subsidy $P_F(p)$ can be implemented through a reduction of rates, so that the firm net repayment, if successful, is $R(p) - P_F(p)$. This

could be a reimbursement to the firm or to the bank which in the latter case is conditional on the bank offering the rate $R(p) - P_F(p)$. Notice that a credit guarantee contract with the firm (rather than with the bank) would have a negative impact on the moral hazard constraint. Indeed, it would increase the attractiveness of the private benefits and low probability of success project, because if the project fails, the firm will still obtain a positive profit.

4 Competition and Credit Market Equilibrium

So far, we have simplified the analysis by assuming an exogenous loan rate, $R(p)$, but, of course the market equilibrium may imply that this rate is itself affected by subsidies to lending, and it may be the case that part or all of the subsidy is passed down to firms. To deal with these concerns, we now study the market equilibrium and its implications for optimal subsidies to loans.

Modeling credit market competition in an imperfect screening framework requires obtaining the optimal interest rate and screening level setting strategies. Since it is not our objective to innovate in the modeling of competition in an imperfect screening framework, we follow here the "classical" approach of Broecker(1990) and Ruckes(2004).

In this framework, because screening is not costly to firms, firms will simultaneously apply to all banks. Each bank will then screen all the firms and make offers to those that are revealed to be good. Because signals convey perfect information they are perfectly correlated across the banks that obtain a signal. Good firms may receive more than one offer and will then choose to borrow from the bank that offers the lowest interest rate. Because of the absence of capacity constrains,undercutting the competitors rates is always profitable, and this leads to the absence of a pure strategy equilibrium, contrary to other approaches (as in Freixas et al., 2007).

Assume N banks are active in the market. The probability of a bank j, $j \neq i$, not granting a loan to a good firm will be the probability of either getting a good signal but setting too high a repayment or getting no signal, which occurs with probability $1 - q$. Restricting the analysis to the symmetric equilibrium case, for bank i to be able to grant a loan, it has to be the case that the $N - 1$ other banks j are either quoting larger repayments or obtained no signal. Thus, the probability of granting a loan at rate R_i to a good firm is $[q(1 - F(R_i)) + 1 - q]^{N-1}$.

Consequently, when quoting R_i, a bank i confronted with $N - 1$ competing banks will have an expected revenue[14] equal to:

$$\Pi(R_i) = \mu q I(pR_i - 1) \left[q(1 - F(R_i)) + 1 - q\right]^{N-1} \tag{23}$$

Because in a mixed strategy equilibrium all strategies yield the same expected profit, the equality $\Pi(R_i) = K$ allow us to obtain the common cumulative

[14] Because the banks signals are perfect, they are perfectly correlated, and once a firm is known to be good, it is known to be good for all banks receiving the signal.

probability distribution $F(R)$, that satisfies $K = \mu qI(pR_i - 1)\left[q(1 - F(R)) + 1 - q\right]^{N-1}$

The repayment R_i is bounded below by the zero profit lower bound, $R_i \geq \frac{1}{p}$ and above by the pledgeable cash flow $y - \frac{B}{I\Delta p}$ that we denote by y'. Because this upper limit is a possible strategy that satisfies $1 = F(y')$ we have:

$$K = \mu qI(py' - 1)\left[1 - q\right]^{N-1}$$

so that, using the assumption of a symmetric equilibrium, condition (23) can be rewritten as $(py' - 1)\left[1 - q\right]^{N-1} = (pR - 1)\left[q(1 - F(R)) + 1 - q\right]^{N-1}$

From which $F(R)$ is obtained

$$F(R) = \frac{1}{q}\left\{1 - (1 - q)\left[\frac{py' - 1}{pR - 1}\right]^{\frac{1}{N-1}}\right\}$$

Denote by $\underline{R}$ the lower bound for R_i, which is the solution to $F(R) = 0$. Thus, $\underline{R}$ satisfies $1 = (1 - q)\left[\frac{py' - 1}{p\underline{R} - 1}\right]^{\frac{1}{N-1}}$ so that

$$p\underline{R} = 1 + (1 - q)^{N-1}(py' - 1)$$

Remark 6 *The solution therefore leads to positive profits* $\mu qI(1-q)^{N-1}(py' - 1)$ *even for the lowest bound* $\underline{R}$*, provided* $q < 1$*.*

Remark 7 *Notice that banks' per dollar profits are larger than their average costs because of the convexity of* $C(q)$*, so that banks participation constraint is always satisfied.*

Remark 8 *Banks will quote repayments* R *in the range* $(\underline{R}, y')$*, and good firms will choose the best offer, provided they have at least one offer, which occurs with probability* $1-(1-q)^N$*. The spread of prices depends upon the difference* $y' - \underline{R}$*, which, itself depends upon* p*. Firms with* $y' < \underline{R}$ *will receive no offer as they would have no incentives to choose the right project. Replacing* $\underline{R}$ *by its value, we observe* $y' < \frac{1+(1-q)^{N-1}(py'-1)}{p}$ *is equivalent to* $y' < \frac{1}{p}$*.*[15] *Not surprisingly, it is risky firms that will be rationed because of moral hazard.*

4.1 Equilibrium Screening Level

Given this equilibrium pricing strategy, it is easy to obtain the optimal level of screening in the absence of a subsidy. The bank maximizes

[15] If $py' < 1 + (1 - q)^{N-1}(py' - 1)$, then

$$(1 - q)^{N-1}(py' - 1) > py' - 1$$

But this implies

$$\left[(1 - q)^{N-1} - 1\right](py' - 1) > 0$$

Because $\left[(1 - q)^{N-1} - 1\right] < 0$, the condition is equivalent to $py' - 1 < 0$

$$\max_{\widehat{q}} \int_{\underline{R}}^{y'} \Pi(R_i) dF(R_i) - C(q)$$
$$\widehat{q} \leq 1$$

But, because $\Pi(R_i) = K = \mu q I(py' - 1)\left[1-q\right]^{N-1}$ (with y' equal to the firm's pledgeable income) the problem is simplified and only an interior solution exists, that satisfies

$$\mu I(py' - 1)(1-q)^{N-1} = C'(\widehat{q}) \tag{24}$$

where $\widehat{q}$ is the bank's optimal screening level given other banks' screening q. Notice that $q = 1$ will never hold in a symmetric equilibrium. Consider, as an example, the case of linear screening costs, $C\left(q\right) = \alpha q$. While in absence of competition screening in this case, if any, is $q = 1$,the market solution with competition implies $(1-q)^{N-1} = \frac{\alpha}{\mu I(py'-1)}$, so that $q < 1$. The equilibrium level of screening decreases with N. Despite this fact, more firms may end up screened because, for any given q,, the likelihood of being served by at least one bank grows with the number of banks.

Remark 9 *It is interesting to observe the connection between firms' moral hazard and screening, because at the limit point $y' = \frac{1}{p}$ equation (24) implies the screening level is zero, so that banks will not lend anyway. The linear screening cost example makes this connection clear: the equilibrium screening level is directly related to the pledgeable income.*

Remark 10 *Because $(1-q)^{N-1}$ is decreasing in N, the impact of increased competition, due to a larger number of banks N on the symmetric equilibrium ($\widehat{q} = q(N)$) is to decrease $q(N)$. Still, since the measure of firms that are financed is $(1-(1-q)^N)$, the overall effect of competition is to improve the efficiency of credit allocation.*

This result is in line with the finding of Dick and Lehnert(2000), which imply that deregulation and fiercer competition, increased access to credit but does not reflect their finding that bank loan losses decrease, as we take the screening technology as given.

Remark 11 *From equation (24) it is easy to derive the impact of changes in the other banks' screening level q on the bank optimal level $\widehat{q}$ and show that banks' screening strategies are strategic substitutes.*

4.2 Optimal Subsidy Policy

In order to determine how competition changes the optimal subsidies, recall, first, that the probability of a good firm not being granted credit is $(1-q)^N$, so that the probability of a firm being financed in equilibrium is $\mu(1-\ (1-q)^N)$.

Second, only firms such that $y' < \frac{1}{p}$ are susceptible of receiving a subsidy P_F. In addition, it is clearly inefficient to leave a rent to the firm above $\frac{B}{\Delta p}$. As a consequence, if a firm receives a subsidy P_F, the subsidy will satisfy $y' + P_F(p) = \underline{R}$. That is, of the support of interest rates that constitute the banks' mixed strategy competition equilibrium, $\underline{R}$ (but not higher interest rates) will be made feasible by the subsidy to firms.[16] This implies the mixed strategy distribution becomes a pure strategy.

The PDB problem may now be written:

$$\begin{aligned}
&\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \left\{\mu I \left[1-(1-q(p))^N\right][py-1-\lambda(S_C(p)+pP_F(p))] - NC(q(p))\right\} f(p)dp \\
&\mu I(1-q)^{N-1}(py'+S_C-1) - C'(q(p)) = 0 \qquad (25)\\
&p\left(y'+P_F(p)\right) \geq 1 \\
&S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p);
\end{aligned}$$

As before, denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraint (25) and to the moral hazard constraint, and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.The Lagrangian conditions become

$$\begin{aligned}
-\lambda(1-(1-q(p))^N)f(p) + (1-q)^{N-1}\nu(p) &\leq 0 \\
-\lambda\mu I(1-(1-q(p))^N)f(p) + \gamma(p) &\leq 0 \\
\mu I N(1-q(p))^{N-1}(py-1-\lambda\left(S_C\left(p\right)+pP_F\left(p\right)\right)) - NC'(q\left(p\right)) - & \\
-\frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} &= 0 \\
\mu I(1-(1-q(p^*))^N)(p^*y-1-\lambda(S_C(p^*)+pP_F(p^*))) - NC(q(p^*)) &= 0
\end{aligned}$$

For firms with $p > \frac{1}{y'}$ it is optimal to set $P_F = 0$, and because $q < 1$, we have, for $S_C > 0$ and $\nu(p) = \frac{\lambda(1-(1-q(p))^N)f(p)}{(1-q)^{N-1}}$:

$\mu I\left[(1-q(p))^{N-1}(py-1) - \lambda S_C\left(p\right)\right] - C'(q\left(p\right)) - \lambda\frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}}C''(q(p)) = 0$

Substracting (25) leads to

$$\mu I\left[(1-q(p))^{N-1}\left(p(y-y') - (1+\lambda)S_C\right)\right] - \lambda\frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}}C''(q(p)) = 0$$

So that the optimal subsidy satisfies

[16] Recall that, for industries with p such that $y' < \frac{1}{p}$, no equilibrium with screening will be feasible in absence of S_F.

$$S_C = \frac{1}{1+\lambda}\left\{ p(y-y') - \frac{\lambda \frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}} C''(q(p))}{\mu I (1-q(p))^{N-1}} \right\} \tag{26}$$

For firms with $p < \frac{1}{y'}$ a subsidy $P_F > 0$ satisfying $p(y'+P_F) = 1$ is enough for the firm to choose the good project. Substituting into the first order conditions, we obtain:

$$S_C = \frac{1}{1+\lambda}\left\{ p(y-y') - \lambda(1-py') - \frac{\lambda \frac{(1-(1-q(p))^N)}{N(1-q(p))^{N-1}} C''(q(p))}{\mu I (1-q(p))^{N-1}} \right\}$$

Firms will receive subsidies, if any, when they satisfy both $py' < 1$ and $p\epsilon(p^*, \frac{1}{y'})$, but at an interest rate that implies a zero profit for the bank and $q(p) = 0$, so that the subsidy to the firm is ineffective if not accompanied by a credit subsidy.

Consider again the example of linear screening costs, $C(q) = \alpha q$, and assume $p > \underline{p}$, so that $P_F = 0$. Optimal policy implies $S_C = \frac{p(y-y')}{1+\lambda}$ and an increased level of screening with respect to the market solution, given by $(1-q)^{N-1} = \frac{\alpha}{\mu I\left(py' + \frac{p(y-y')}{1+\lambda} - 1\right)}$. The link that competition introduces between screening and moral hazard–highlighted in remark 9 is evident again in the fact that, even with $p > \underline{p}$, the optimal subsidy depends on pledgeable income y', since the support of equilibrium interest rates depends on y'. The convex costs case, $C = \frac{\beta C^2}{2}$, meanwhile, yields $S_C = \frac{\lambda\beta}{\mu I(1+\lambda)}\left(\frac{1-(1-q)^N}{N^2} - \frac{N^2(1-q)^{N-1}}{N^2}\right)$. The number of banks, N, has an ambiguous effect on S_C: while greater competition reduces the effectiveness of the subsidy to increase banks incentives to screen–because part of the subsidy is passed on to firms via reduced prices–, competition also increases the need for the subsidy.

Regarding the implementation, it is sufficient to provide funding at below the market rate. Requiring banks to set low interest rates on loans would be inefficient in terms of increasing screening levels. Indeed, commercial banks would react by undercuting the low interest rates on loans, so that the distribution of interest rates will shrink and concentrate on lower rates. While this could be interesting as it promotes cheaper funding, it is unrelated to the objective of increasing banks' incentive to screen.

5 Extensions: collateral, liquidity and capital shortages.

So far, we have considered subsidies to banks and firms in a market where firms cannot pledge collateral and banks are able to issue any type of liability and face no constraint, either on their liquidity or on their solvency. When this is not the

case, the analysis of optimal subsidy policy changes. To simplify the analysis, we assume away the mixed strategies characteristic of competition, and assume each bank faces its own market repayment.

5.1 Collateral

To begin with, a preliminary remark on the difference between collateral and PDB credit guarantees is in order. Although in both cases the bank will recover a fraction of the loan in case the borrower defaults, in the collateral case it affects the borrower itself with completely different implications on the incentives. Because the borrower is not affected by public credit guarantees, their existence will increase the banks' expected return and, therefore, it will also increase its screening level. As mentioned, provided (16) is satisfied, as screening is determined by expected profit and not by the difference in the payout between good states and bad states (that is characteristic of moral hazard problems), credit guarantees play the role of a subsidy to lending. Collateral, instead will play a key role in the firms' self selection that may be a substitute for screening as we will now develop.

So far, we have assumed that a bank receiving no signal on a firm will not finance it. Nevertheless, this need not be the case if the firm is to post collateral[17]. In this case, however, it is possible that the amount of the loan the firm obtains is constrained by the availability of collateral and the firm's project has to be downsized. We extend now our analysis to the case where agents are endowed with some exogenously given amount of collateral, which we denote V[18].

As it is standard, we will assume collateral is costly, as the V value of the asset to the bank is lower than its value to the firm, $(1+\delta)V$, where $\delta > 0$. In the present setup, collateral will play two related roles: as a signalling device and in mitigating credit risk.

Signalling allows good firms to separate from bad firms, if the latter are not willing to post collateral. Let $R_V(p, V)$ be the per dollar repayment on a loan I collateralized with an asset valued V to the bank. Because firms know their types[19], when the value of collateral V is larger than some threshold, only good firms will be ready to pledge their collateral. Define ν_B as the collateral per dollar of loan that leaves the bad firms indifferent between a partially collateralized loan and abstaining from applying for a loan. That is, ν_B satisfies the following condition:

$$p_(y - R_V(p, V)) - (1 - p_)(1 + \delta)\nu_B = 0$$

[17] If property rights do not provide legal certainty to pledging and repossession, however, collateral based credit may be quite limited.

[18] This amount will depend, among other factors, upon the legal and institutional features of the economy.

[19] If firms do not know their type, under our assumption of an expected negative present value for the average firm, $(\mu p + (1-\mu)p_)y < 1$, if banks break even, firms will make losses and, therefore will abstain from asking for a collateralized loan.

Then, any loan contract with a collateral to loan ratio $\frac{V}{I}$ that satisfies $\nu_B \leq \frac{V}{I}$ will deter bad firms from applying for a loan. Because downsizing has an opportunity cost for the firms, efficient contracts will be characterized by the maximum loan per unit of collateral, that is the minimum $\frac{V}{I}$ that satisfies $\nu_B \leq \frac{V}{I}$.This implies the good firm individual rationality constraint is trivially satisfied, for any contract characterized by a collateral to loan ratio v_B. This ratio, jointly with V will determine the maximum size I at which the firm will be able to develop its project.

Notice that whenever the above inequality is satisfied it is unnecessary for banks to screen firms for collateralized lending. The use of collateralized loans implies that all good firms have their projects funded so that there is no credit rationing due to banks' insufficient screening.

Still, depending on the availability of collateral V and on the cost $(1-p)\delta$ of pledging it, the firm may prefer to be screened by the bank. This will be the case if the firm's profits are higher with an uncollateralized loan, that is:

$$p(y - R(p))I^* > p(y - R_V(p))\frac{V}{\nu_B} - (1-p)(1+\delta)V$$

where I^* is the size of the loan required to finance the project without downsizing. The condition is obviously met when collateral is scarce. Still, even if collateral is plentiful, if its cost δ is sufficiently high in comparison to the cost of screening, the condition is also fulfilled[20]. In the following we will assume the condition is so that both firms and banks prefer to screen, so that banks' screening and public support to firms are still an issue. Notice, though, that when this condition is not satisfied, and the firm prefers to borrow collateralized because it has sufficient collateral, the policy implication is clear: the PDB should abstain from any intervention.

Because it is efficient to screen firms, while collateral lending has no cost, it seems natural that first banks invest in screening, but if no signal is obtained, they offer the firm the possibility of a smaller collateralized loan that is only attractive to good firms. When this is the case, the objective function of the bank is modified. If the bank obtains a non-informative signal, which occurs with probability $(1-q)$, it will still be able to grant a collateralized loan. The bank profits will now become:

$$\max_q \mu \left\{ q(pR(p) - 1)I^* + (1-q)\left[(pR_V + (1-p)\nu_B) - 1\right]\frac{V}{\nu_B} \right\} - C(q))$$

[20] Because in equilibrium per dollar expected profits should be equal across banks, we have $pR(p)) - \frac{C(q(p))}{\mu I} = pR_V(p) + (1-p)\nu_B$

A sufficient condition for the above inequality to be satisfied is:

$$\frac{C(q(p))}{\mu I} \leq (1-p)(1+\delta)\nu_B$$

because, in this case, the firm prefers an uncollateralized loan, even in the absence of any downsizing, simply because the expected cost to the firm of losing its collateral is higher than the screening cost to the bank.

Still, this is only an extreme sufficient condition when, in fact, downsizing has an opportunity cost that makes our hypothesis of efficient screening even more natural.

The first order condition that determines the level of screening will be:

$$\mu \left\{ (pR(p) - 1)I^* - [(pR_V + (1-p)\nu_B) - 1] \frac{V}{\nu_B} \right\} = C'(q))$$

Consequently, the introduction of collateral decreases q through the "spare tire" effect of collateralized lending when the bank obtains no signal. Of course, this does not mean that a policy promoting the use of collateral by protecting creditors' rights to repossession should not be implemented. It simply states that it has a cost in terms of relationship banking and in the lower level of screening it generates. The result is in line with Manove et al.(2001) model of "lazy banks" and has competition policy and regulatory implications. Indeed, on the competition policy side, it implies that the lower the banks' market power in the collateralized market, $pR_V + (1-p)\nu_B - 1$, the higher the level of screening in the uncollateralized segment. On banking regulation, it implies that collateralized loans should have very low capital charge, in line with Basel II and III, and excess of caution will be costly in terms of screening incentives.

Thus, overall, the introduction of collateralized lending will, on the one hand, increase the total output but, on the other hand, diminish the bank's incentive to screen.

Because the subsidy in case of a collateralized loan is not justified, the second best problem becomes:

$$\begin{aligned}
&\max_{S_C(p), P_F(p), q(p), p^*} \int_{p^*}^{1} \{\mu \left[q(p)(py-1)I^* + (1-q(p)) \left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V \right) \right] \\
&-C(q(p)) - \lambda\mu q I^*(S_C(p) + pP_F(p))\} f(p)dp \\
&\mu \left[(pR(p) + S_C(p) - 1)I^* - [(pR_V + (1-p)\nu_B) - 1] \frac{V}{\nu_B} \right] - C'(q(p)) = 0 \\
&[y + P_F(p) - R(p)] I^* \Delta p \geq B \qquad (27) \\
&S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p);
\end{aligned}$$

Denote, as before, by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to the first two constraints, and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_C(p), P_F(p), q(p)$ and p^* are:

$$-\lambda q f(p) + \nu(p) \leq 0 \tag{28a}$$

$$-\lambda \mu q f(p) + \gamma(p)\Delta p \leq 0 \tag{28b}$$

$$\mu[(py-1)I^* - \left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right) - \lambda\left(S_C(p) + pP_F(p)\right)]I^*$$
$$-C'(q(p)) - \frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} = 0 \tag{28c}$$

$$\mu\left[q(p^*)(p^* y(p^*) - 1)I^* + (1 - q(p))\left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right)\right]$$
$$-C(q(p^*)) - \lambda I(S_C(p^*) + pP_F(p^*)) = 0$$

The analysis of subsidies to screened firms is the same as before.[21]

$$p(y - R_V\frac{V}{\nu_B} - (1-p)(1+\delta)V (\geq (p-\Delta p)(y - R_V\frac{V}{\nu_B} - (1+\Delta p - p)(1+\delta)V + B$$

This implies

$$R_V \leq y + (1+\delta)\nu_B - \frac{B\nu_B}{V\Delta p} \tag{29}$$

with, as intuition suggests, a much higher pledgeable cash flow due to the value of collateral to the firm. The main impact of collateral will be on the optimal credit subsidy, as the option of collateralized lending decreases the benefits of screening.

Following the same procedure that we used to derive the optimal subsidy in the absence of collateral, we obtain, when a collateral V could be pledged with the bank:

$$\begin{array}{lll} S_C(p) & = & \{p(y - R(p)) - \left(p(y-R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*} \\ -\frac{\lambda q}{\mu I^*}C''(q(p))\}\frac{1}{1+\lambda} \text{ if } q(p) < & 1 & \\ S_C(p) & \leq & \{p(y - R(p)) - \left(p(y-R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*} \\ -\frac{\lambda}{\mu I}C''(1)\}\frac{1}{1+\lambda} \text{ if } q(p) = & 1 & \end{array} \tag{30}$$

This expression allows us to identify the industries that should be targeted, that is, such that $S_C(p) > 0$. These industries will be characterized by the following expression:

$$p(y - R(p)) \geq \left(p(y-R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*} + \frac{\lambda q}{\mu I}C''(q(p)) \tag{31}$$

[21] The introduction of collateral, however, also modifies the moral hazard problem for firms receiving collateralized loans, as they will now choose the positive net present value project taking into account the possible loss of collateral:

For a given expected profit, our findings square with the argument that firms lacking the possibility of collateralizing their loans are desirable targets of public financing. In particular, low available collateral V and high minimum required collatera lν_B make it more likely that the above condition is fulfilled. Small and young firms, and those in sectors holding little pledgeable assets (such as services), are likely examples of such targets.

The comparison between the level of the subsidy when there is no collateral, (9) and when there is collateral, (30) shows that the subsidy is much larger in the first case. The explanation is obviously that, in the first case, the social loss of the bank not getting any signal is the loss of $p(y-1)$, while in the second case, the cost is only a lower level of funding for the firm, corresponding to $p(y-1)(I-V(1-\gamma))$,which depends upon the amount of collateral V available at the firm level. Of course, this does not mean that a policy promoting the use of collateral by protecting creditors' rights to repossession should not be implemented. It simply states that it has a cost in terms of relationship banking and in the lower level of screening it generates.

5.2 Liquidity

The decrease in the volume of credit that characterizes a crisis may result from a decrease in its demand or in its supply. In the second case, it may be due to a reduction in banks' access to funding. In our set up, the banks limited access to funds can be easily modelled through the introduction of an additional constraint limiting the bank's total credit supply in the analysis of the second best[22]. Implicitly, it is assumed that the supply of outside liquidity by monetary policy authorities cannot be altered, and that the PDB is not forced to support the monetary contraction policy (although a reinterpretation of λ could account for the PDB liquidity shortage) and is able to pursue its own lending policy.

In such a framework the banks choice of screening will take the constraint into account, as they will now maximize

$$\max_{q(p),p^u} \int_{p^*}^{1} \left\{\mu q(p)(pR(p)+S_C(p)-1)I - C(q(p))\right\} f(p)dp$$
$$\int_{p^*}^{1} \mu q(p) I f(p)dp \leq L \tag{32}$$

The solution to this problem will be, if ϕ is the Lagrangian multiplier associated to the liquidity constraint:

[22] This, of course, disregards why and how a liquidity shortage occurs. Considering those reasons would require the modeling of the whole monetary policy framework.

$$\mu(pR(p) + S_C(p) - (1+\phi))I - C'(q(p)) = 0$$
$$\int_{p^*}^{1} \mu C'^{-1}(\mu(pR(p) - (1+\phi))I))If(p)dp \leq L$$
$$\mu q(p^*)(p^*R(p^*) - (1+\phi))I - C(q(p^*)) = 0$$

Not surprisingly the liquidity restriction implies a shadow cost of liquidity that can be interpreted as an interest rate increase.

The PDB will now solve:

$$\max_{S_C(p),P_F(p),q(p),p^*} \int_{p^*}^{1} \{\mu q(p)(py-1)I - C(q(p)) - \lambda\mu qI(S_C(p) + pP_F(p))\} f(p)dp$$
$$\mu(pR(p) + S_C(p) - (1+\phi))I - C'(q(p)) = 0$$
$$\int_{p^*}^{1} \mu C'^{-1}(\mu(pR(p) - (1+\phi))I))If(p)dp \leq L$$
$$\mu q(p^*)(p^*R(p^*) + S_C(p^*) - (1+\phi))I - C(q(p^*)) = 0$$
$$[y + P_F(p) - R(p)]\, I\Delta p \geq B$$
$$S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p);$$

Now, depending on the way the subsidies are implemented, they may imply additional liquidity. Under intermediated lending, the PDB will be able to use, in fact two instruments: $S_C(p)$ and $\Delta L(p)$, a credit line that will alleviate the liquidity constraint for loans in the industry p and the liquidity constraints.[23]. Let $\phi(p)$ be the Lagrangian multiplier associated to the liquidity constraint (32).

It is easy to prove that, if $\phi > 0$, that is, if the liquidity constraint is binding, the use of $\Delta L(p)$ will always strictly improve upon the exclusive use of subsidies implemented through instruments unrelated to liquidity. Indeed, assume, by way of contradiction, that the optimal structure constrained by $\Delta L(p) = 0$ is obtained. Because in the constraints of the above problem, only the expression $S_C(p) - (1+\phi)I$ appears, it is clear that a positive $S_C(p)$ can be substituted by the equivalent decrease in $\phi(p)$ that is generated by an increase in $\Delta L(p)$. Still, while $S_C(p)$ has a λ cost, an increase in liquidity has no cost. So, even if the optimal policy may still involve a subsidy, it will be combined with a policy of intermediate lending that will alleviate the bank's liquidity constraint and thus reduce the opportunity cost of lending for some specific industries p.

5.3 Capital Shortages

The banks' lack of regulatory capital, characteristic of a credit crunch (See Bernanke and Lown, 1991) may also impose a limit to the banks' ability to lend.

[23] The issue of firms' rationing on their long term funding, which we consider quite relevant, is beyond the scope of our analysis.

Although the equation that captures the restriction is similar to the liquidity constraint (32)above? the effects will be quite different. Denote as β the risk weight associated to firms' lending, that is, the coefficient of required capital to extend a given amount of credit. If all the loans to firms have the same risk weight, the constraint will be:

$$\int_{p^*}^{1} \beta \mu q(p) I f(p) dp \leq E$$

If capital shortages are a key constraint, then reducing the loss given default on a loan is a feasible way to soften that constraint. Because a credit guarantees program reduces the banks' risk for the targeted loans as exposure is reduced from I to a fraction $(1-\frac{G(p)}{I})I$, if $\frac{G(p)}{I}$ is the fraction of losses the PDB commits to cover. This means that the PDB credit guarantees program is the right way to intervene. Nevertheless, the impact of credit guarantees will depend upon the rating of the PDB. With an ill-rated PDB, credit guarantees by the PDB may not be credible and therefore be ineffective.

6 Business and Credit Cycles

An important issue regarding the activity of a PDB is its role in a situation where banks are constrained in their lending, and direct public financing is expected to play a particularly important countercyclical role (Luna-Martinez et al., 2012). Our framework provides a rationale for this expectation as recessions may be times of particularly acute liquidity and capital restrictions for the banks, specially when associated with financial crises. For this reason, funding to banks and credit guarantees will help ease liquidity and capital constraints as established in section 5, and will be particularly valuable during times of crises. However, crises may also be times when expected return from new projects is particularly low (low py), reducing, for any given $R(p)$, the externality that implies underprovision of screening, and increasing incentives to engage in moral hazard. Our analysis thus suggests that, if the decrease in the volume of credit is demand driven, PDB interventions should be reduced. Still, if, as it seems more likely, the reduction is due to liquidity and capital constraints, the PDB will play a key countercyclical role. Lending to banks is optimal only to the extent that there are starker liquidity constraints associated with the crisis. If, instead, the reason for the reduction in credit is banks' capital constraints, then credit guarantees with the corresponding reduction of the banks' exposures will be the correct way to intervene. So, it is optimal for the PDB to offer both lending and credit guarantees so that banks themselves will choose the type of suppport they prefer for the same implicit level of S_C, depending on the constraint they face. Bear in mind, however, that our static framework is not well suited to deal with dynamic costs from crisis in the presence of credit constraints. [24]

[24] For instance, Eslava et al. (2015) estimate that there are long-lasting TFP losses from the inefficient exit of profitable but credit constrained firms.

The empirical evidence shows that macroeconomic conditions have a strong impact on credit, with tightening standards associated with lower future levels of loans and output (Lown and Morgan, 2001 p.1581). In the context of our model, this means either that banks screen more in bad times or that, in good times, banks lend indiscriminately(Ruckes, 2004). The first is possible when, in a downturn, the supply of credit decreases more than the demand leading to higher $R(p)$, and therefore a higher screening effort q. If this is the case, the implication is that subsidies should decrease in bad times. Still, liquidity and capital shortages imply, instead that the PDB develops a more active program of intermediated lending and credit guarantees, even in the absence of a subsidy. So, there is no one unique strategy for the PDB and a diagnosis is required before deciding the cure.

Consequently, the activity of a PDB may switch with the business cycle. In normal times its role will be the one described in our model, providing banks with the incentives to increase their screening and, therefore, their lending to high potential ($p(y - R(p))$ industries. In a downturn or in a crisis, instead, the liquidity and solvency constraints may be more important, and, in this case the PDB can reduce its subsidies and concentrate on providing liquidity at the market price or providing a credit guarantees insurance at its fair value, which is equivalent to writing a credit default swaps. It is important to point, however, that our model abstracts from public lending for working capital, which may be crucial in a downturn (Eslava et al. 2015).

7 Robustness

At this stage is is interesting to examine how robust are our qualitative results.

Regarding the screening technology, our framework treats screening as weeding out bad firms. Would the same results hold if, instead we had a screening technology based on an imperfect signal? The answer is affirmative provided screening is costly.

- **Screening technology**

In this case, screening will provide a signal s on the firms' distribution of cash flows y, generating an ex post distribution with density function $f(y \mid s)$,which is informative about y in the sense of the Monotone Likelihood Ratio Property (MLRP), so that high signals imply a higher probability mass on the high cash flows. When this is the case, the optimal decision for the bank will be to lend whenever the signal is higher than some threshold s^*. The bank choice of screening corresponds then to the precision of the signal s, ranging from a perfect signal $y = s$ at a high cost to no precision at all (in which case $f(y \mid s) = f(y)$) at zero cost. The precision level will result from profit maximization and, again, will not take into account the benefits accruing to the firms of the choice of precision, $p(y - R(p))$.

Still, the analysis of competition will lead to different conclusions, because, signals will not be perfect any longer, so that bad firms will have a chance to be

granted credit. This implies, as in Broecker(1990) that when the population of banks increase the chances of bad firms to obtain credit increases, so that for a given interest rate, the average return on a bank loan may decrease.

- **Firms' Moral Hazard**

Regarding firms' moral hazard, we have assumed a unique solution$\underline{p}$ to the equation $y - R(p) = \frac{B}{\Delta p}$, but the extension to a more general case even if more cumbersome is straightforward. It will define N intervals $(p_1,p_2),...(p_{2N-1},p_{2N})$ such that for any $p, p\epsilon(p_{2k-1}, p_{2k}), y - R(p) < \frac{B}{\Delta p}$. Then our proof extends and it is possible to prove that it is always beneficial to subsidize firms $p_{2k-1} + \varepsilon$ and $p_{2k} - \varepsilon$ for ε sufficiently small, as a very limited subsidy allows $q(p)$ firms in this interval to be financed and generate $\mu q(p)(py - 1)$ additional output which is independent of ε. Other forms of moral hazard, as firm's effort level could be considered. In the appendix we briefly examine an alternative modeling of moral hazard, through the introduction of a cost of effort function at the firm level and the implications it would have, and show that, again, it would be optimal to subsidize firms with insufficient incentives to exert effort.

- **Loan Size**

We have assumed that the screening cost does not depend upon the size of the project and of the loan. This seems a reasonable yet critical assumption. Indeed, if the screening costs were to be proportional to the projects' size, it would imply that size is irrelevant in the screening decision and small firms would have the same chances of being financed as large firms.

Also, we have assumed $R(p)$ does not depend upon the size of the loan. This implies the bank choice of $q(p)$, when confronted with a repayment $R(p)$, a subsidy, $S_C(p)$, and a size $I(p)$ will result from the first order condition:

$\mu(pR(p) + S_C(p) - 1)I(p) - C'(q(p)) = 0$, with the simplification that it is the marginal cost of screening per dollar of granted loan $\frac{C'(q(p))}{\mu I(p)}$ that matters. Dropping the assumption would simply imply that the bank optimal screening level will result from the total revenue $R(p, I(p))$ and total subsidy $S_C(p, I(p))$. The impact of size will then cease to be linear, but the qualitative results would remain the same.

- **Industry Specific Screening Costs**

Finally, it is often argued that screening might be more or less costly in different industries. This is the case, for instance, for SMEs. As stated by Beck et al. (2008, p.1-2)"Both high transaction costs related to relationship lending and the high risk intrinsic to SME lending explain the reluctance of financial institutions to reach out to SMEs". In addition, the scarcity of reliable data on SMEs and the possible manipulation of their financial statements make screening more costly. Still, the argument is also true for young firms as well as for young industries. In our model, if repeated lending to the same industry decreases the screening cost, the optimal subsidies should also decrease. When

this is the case, subsidies should be directed to "nascent" industries and should disappear from "senescent" industries.

Finally, if the screening cost is related to relationship lending, then a high turnover in the population of firms make the investment in the relationship less profitable. In our context, this implies considering a screening function $C(q(p), p)$, which is a straightforward extension.

8 Conclusion

The existence of specific programs of credit to firms has sometimes been justified on the basis of the positive externalities they generate or on the existence of moral hazard at the firm level. We argue that this justification of public credit support is, in fact, unrelated to the credit market and could be dealt with through a direct subsidy. Our focus is instead on the welfare costs of financial markets imperfections. We argue that, when the screening of projects is costly, banks best strategy is to set their screening levels as a function of their expected profit on the operation, which results in some good firms being credit rationing. Still, the banks profit maximizing level of screening is suboptimal because it disregards the profits the financing of a project generates at the firm level. To correct for this underinvestment in screening, a PDB can intervene in a number of ways that may vary depending on the constraints banks face. Nevertheless, the firms the PDB should target industries characterized by:

1. Some degree of credit rationing
2. A sufficiently high expected firms' profits (i.e. high $\mu p\left(y - R\left(p\right)\right)$), as this reflects the benefits of screening that are not internalized by the bank.
3. Projects with sufficiently large financing needs I and with a high proportion of good firms μ .

This implies he PDB has to have access to information on the industries, so as to identify these characteristics, an information that may be facilitated by the existence of credit registries.

Regarding the way a PDB can instrument its intervention, if there is no bias in the PDB objective function, direct lending is the preferred way. Nevertheless, as the empirical evidence has shown that lack of rigorous corporate governance, government pressures and political biases makes direct lending inefficient, intermediated loans may be preferred. We show how the PDB can improve efficiency through either subsidized lending or credit guaranties as they are equivalent in good times. Still, when banks face a liquidity or capital shortage, the two programs can be adapted so as to react to the more pressing constraints.

Finally, the availability of collateral should also be taken into account. While the creation of legal certainty on collateral provides better access to the credit market, we show that it also reduces banks incentives to screen and may lead

banks to reduce the size of the loans to firms, forcing them to downsize their projects.

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10 Appendix: Unobservable Firms' Efforts

An alternative common form of moral hazard is the effort model, whereby firms choose the optimal level of effort (normalized to equal the probability of success) given its quadratic cost $C(e) = \frac{e^2}{2\beta}$. Under perfect observability and contractability of effort, a firm receiving a loan would make a repayment $I(1+\rho)$, so that the firm maximizes

$$\max_e epy - I(1+\rho) - \frac{e^2}{2\beta}$$

and the first best effort level $e^* = \beta py$ is obtained. Under moral hazard, the chosen level of effort $\widehat{e}$, for a repayment $R(p)$ will be the solution to:

$$\max_e ep\left(y - R(p)\right) - \frac{e^2}{2\beta}$$

so that $\widehat{e} = \beta p\left(y - R(p)\right) < \beta py$, where $\widehat{e}pR(p) = I(1+\rho)$

Consequently, a subsidy in conditional on success changes the objective function to $\max_e ep\left(y + P_F(p) - R(p)\right) - \frac{e^2}{2\beta}$ and its solution to

$$e = \beta p\left(y + P_F(p) - R(p)\right) \tag{33}$$

The equivalent second best problem will then have the added variable e and a different moral hazard constraint (33).

$$\begin{aligned}
&\max_{S_C(p),P_F(p),q(p),e,p^*} \int_{p^*}^{1} \left[\mu q(p)(epy-1)I - C(q(p)) - \lambda\mu q(p) I(S_C(p) + epP_F(p))\right] f(p)dp \\
&\mu(epR(p) + S_C(p) - 1)I - C'(q(p)) = 0 \qquad (34)\\
&e = \beta p\left(y + P_F(p) - R(p)\right)\\
&S_C(p) \geq 0; \qquad P_F(p) \geq 0; \qquad 1 \geq q(p); \qquad e \leq 1
\end{aligned}$$

Denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraints (34) and (33), and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_C(p), P_F(p), q(p), e$ and p^* are:

$$-\lambda q f(p) + \nu(p) \leq 0 \tag{35}$$

$$-\lambda\mu qeIf(p) + \beta\gamma(p) \leq 0 \tag{36}$$

$$\mu I\left[epy - 1 - \lambda\left(S_C(p) + epP_F(p)\right)\right] - C'(q(p)) - \frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} = 0 \tag{37}$$

$$\mu q(p) I(py - \lambda pP_F(p)) f(p) + \nu(p)\mu IpR(p) - \gamma(p) = 0 \tag{38}$$

$$[\mu q(p^*)e(p^*y(p^*) - 1) - C(q(p^*)) - \lambda(S_C(p^*) + ep^*P_F(p^*))] = 0 \tag{39}$$

Consider the case $S_C(p) > 0; P_F(p) > 0; 1 > q(p); e < 1$,

$$\begin{aligned} \nu(p) &= \lambda q f(p) \text{ and} \quad (40) \\ \beta p \gamma(p) &= \lambda \mu q e I f(p) \quad (41) \end{aligned}$$

Replacing in 37) and (38) yields, respectively :

$$\mu I \left[epy - \lambda \left(S_C\left(p\right) + epP_F\left(p\right)\right)\right] - C'(q\left(p\right)) - \lambda q C''(q(p)) = 0$$

and

$$\mu q(p) I(py - \lambda p P_F(p)) f(p) + \lambda q(p) f(p) \mu I p R(p) - \frac{1}{\beta} \lambda \mu q e I f(p) = 0$$

dividing by $\mu q(p) I f(p)$ the expression simplifies to

$$py - \lambda p P_F(p) + \lambda p R(p) - \frac{1}{\beta} \lambda e = 0 \quad (42)$$

$P_F(p) \geq 0$ if

$$py + \lambda p R(p) \geq \frac{1}{\beta} \lambda e \quad (43)$$

Expression (43), can simply be interpreted as the benefits of the subsidy being larger than its costs. As the benefits of the subsidy are derived from the incentive effect on e, we want the marginal cost of P_F, $\lambda \mu q I e p f(p)$ to be lower than the benefits it generates. Now, the for each dollar increase of P_F, the impact on e is $\frac{de}{dP_F} = \beta p$. In turn, a unit increase in e will have an direct impact on the objective function of py, and an indirect impact in the incentives for the bank to increase its screening level, because a dollar of $epR(p)$ is equivalent to a dollar increase of $S_C(p)$. So, an increase in e leads to benefits of $py + \lambda p R(p)$, wich occur with probability $\mu q(p) I f(p)$. So the net benefit condition for a subsidty is $\mu q(p) I f(p) \beta p (py + \lambda p R(p)) \geq \lambda \mu q I e p f(p)$. Simplifying we obtain expression (43).

In order to obtain a condition for the positivity of P_F without the endogenous value of e, replacing e by its value $e = \beta p \left(y + P_F(p) - R(p)\right)$ in (42), we obtain:

$$y - \lambda P_F(p) + \lambda R(p) - \lambda \left(y + P_F(p) - R(p)\right) = 0 \quad (44)$$

and

$$P_F = \frac{y(1-\lambda)}{2\lambda} + R(p)$$

So, for $\lambda < 1$, which seems a natural assumption, all firms will be subsidized: the impact on output and the reduction in the cost of subsidizing bank loans are sufficiently strong to yield this result.

Condition $e < 1$ implies, using (33) that $1 > \beta p \left(y + P_F(p) - R(p)\right)$.

If instead, e reaches the corner solution, $e = 1$, and $\beta p\left(y + P_F(p) - R(p)\right) > 1$. This implies $\gamma(p) = 0$, and consequently $P_F(p) = 0$. Consequently, the firms that will receive a subsidy will be those for which $y - R(p) < \frac{1}{\beta p}$, which is the equivalent of $y - R(p) < \frac{B}{\Delta p I}$ in our modeling approach.

Because the moral hazard problem has changed, the firms to which the subsidy will be granted has also changed. While, in the presence of the private benefits switch to private benefits the subsidy was to those firms that had insufficient rents to provide the right incentives (but were close enough), now the subsidy will go to any firm with an effort level lower than $e = 1$.